Exercises

(1) Suppose that for an instance $(G, S = \{s_1, ..., s_k\}, T = \{t_1, ..., t_k\})$ of k-DRP we have a clique subdivision with set of centres $C = \{c_1, ..., c_{3k}\}$ (i.e. there exist a set of paths $\{Q_{i,j} | 1 \leq i < j \leq 3k\}$ such that $Q_{i,j}$ has endpoints c_i and c_j and every pair of these paths intersect only at their common endpoints) and $2k$ vertex disjoint paths from $S \cup T$ to C. Show that the desired $P_1, ..., P_k$ exist.

(2) (a) Show that for any subset S of the vertices of a tree T we can find a vertex v such that no component of $T - v$ contains more than half the vertices of S.

(b) Show that for any such S and v we can partition the vertices of $T - v$ into two sets A and B such that neither A nor B contains more than two-thirds of the vertices of S and there are no edges between A and B.

3) Show that if a 3-connected graph does not have K_4 as a minor then it is a clique of size at most 3.

4) (a) Show that G has a cycle of length l, denoted C_l, as a minor precisely if it contains a cycle of length at least l as a subgraph.

(b) Show that if a non-empty graph G does not have C_l as a minor then it has a vertex of degree at

(c) Let H be the graph consisting of an induced cycle of length 4 and a vertex x joined to every vertex of this cycle. Show that if a 3-connected graph does not have H as a minor then it is a clique of size at most four.

5) Show that if G has $t|V(G)|$ edges and no minor of G other than G has this property then for every vertex v of G, every neighbour of v is adjacent to at least t other neighbours of G.

(b) Use this to show that if a non-empty graph G has $2^{t-1}|V(G)|$ edges then it has a K_t minor.

(6) Suppose that X is a cutset of size 2 in G, and for each component U of $G - X$, the graph G_U obtained from $G[V(U) + X]$(the subgraph of G induced by $V(U) \cup X$) by adding an edge between the vertices of X has no K_t-minor. Show that G has no K_t minor.

(7) Suppose that H is an $l + 1$ connected graph, X is a cutset of size l in G, $G - X$ has at least $l - 1$ components and for each component U of $G - X$, the graph G_U obtained from $G[V(U) + X]$ by adding edges so that X forms a clique has no H-minor. Show that $H \not\leq_M G$.

(8) Show that every 3-connected graph has an edge whse contraction leaves it 3-connected.